Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 2 hours

Paper reference **9MA0/01**

Mathematics

Advanced

PAPER 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1. The point $P(-2, -5)$ lies on the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation

(a) $y = \mathrm{f}(x) + 2$

(1)

(b) $y = |\mathrm{f}(x)|$

(1)

(c) $y = 3\mathrm{f}(x - 2) + 2$

(2)

a) y : +2

(-2, -5) becomes (-2, -5+2) = (-2, -3)

b) Any negative y's become positive

(-2, -5) becomes (-2, 5)

c) x : +2

y : x 3, +2 (order matters)

(-2, -5) becomes (-2+2, 3(-5)+2)

= (0, -13)

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2. $\mathrm{f}(x) = (x-4)(x^2 - 3x + k) - 42$ where k is a constant

Given that $(x+2)$ is a factor of $\mathrm{f}(x)$, find the value of k.

(3)

$(x+2)$ is a factor means that $f(-2)=0$

$f(-2) = (-2-4)[(-2)^2 - 3(-2) + k] - 42$

$= -6(4+6+k) - 42$

$= -6(10+k) - 42$

$= -60 - 6k - 42$

$= -6k - 102$

$f(-2) = 0$

$\Rightarrow -6k - 102 = 0$

$\Rightarrow -6k = 102$

$\Rightarrow k = -17$

P 6 9 6 0 1 A 0 4 4 8

3. A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is furthest away from the origin O,

(b) find the exact length OP

(2)

a)

i) complete the square

$x^2 - 10x + y^2 + 16y = 80$

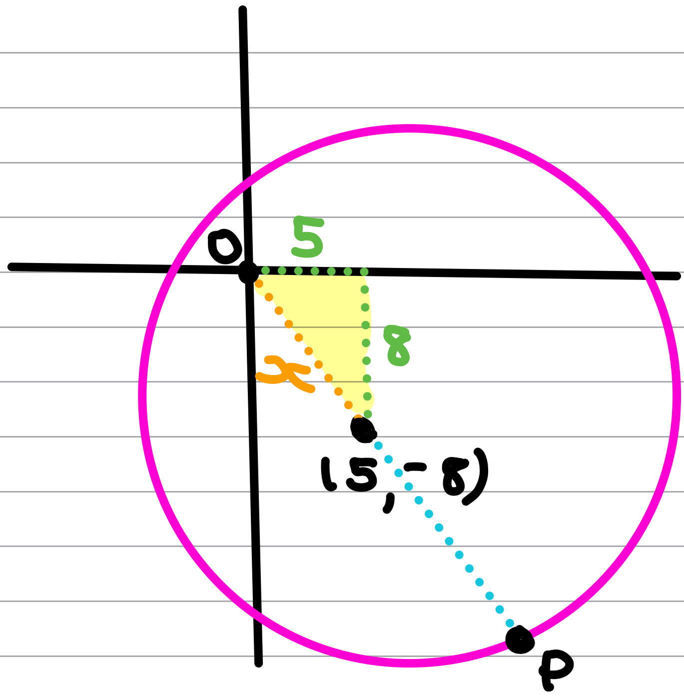
$(x-5)^2 - 25 + (y+8)^2 - 64 = 80$

$(x-5)^2 + (y+8)^2 = 169$

$(5, -8)$

ii) $r = \sqrt{169} = 13$

b)



Look at yellow triangle:

$5^2 + 8^2 = x^2$

$x = \sqrt{25 + 64}$

$x = \sqrt{89}$

$OP = \sqrt{89} + \text{radius}$

$= \sqrt{89} + 13$

$\sqrt{89} + 13$

P 6 9 6 0 1 A 0 6 4 8

4. (a) Express $\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x$ as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x = \ln k$$

where k is a constant to be found.

(2)

a) $\int_{2.1}^{6.3} \frac{2}{x} \, dx$

b) $\left[2\ln x\right]_{2.1}^{6.3}$

$= 2\ln 6.3 - 2\ln 2.1$

$= 2(\ln 6.3 - \ln 2.1)$

use ln rule: $\ln a - \ln b = \ln \frac{a}{b}$

$= 2\ln\left(\frac{6.3}{2.1}\right)$

$= 2\ln 3$

$= \ln 3^2$

$= \ln 9$

$k = 9$

5. The height, h metres, of a tree, t years after being planted, is modelled by the equation

$$h^2 = at + b \qquad 0 \leqslant t < 25$$

where a and b are constants.

Given that

- the height of the tree was 2.60 m, exactly 2 years after being planted
- the height of the tree was 5.10 m, exactly 10 years after being planted

(a) find a complete equation for the model, giving the values of a and b to 3 significant figures. **(4)**

Given that the height of the tree was 7 m, exactly 20 years after being planted

(b) evaluate the model, giving reasons for your answer. **(2)**

$h^2 = at + b$

given:
- $t = 2,\ h = 2.6$
- $t = 10,\ h = 5.10$

a)
$2.6^2 = 2a + b$ ①

$5.1^2 = 10a + b$ ②

solve simultaneously (using elimination or substitution)

Elimination:	Substitution:
$2a + b = 6.76$ ①	$b = 6.76 - 2a$ ①
$10a + b = 26.01$ ②	sub into ②
$-8a = -19.25$	$26.01 = 10a + (6.76 - 2a)$
	$26.01 = 8a + 6.76$
$a = 2.40625$	$8a = 19.25$
sub into either equn. to get b	$a = 2.40625$
$\Rightarrow b = 6.76 - 2(2.406)$	$b = 6.76 - 2(2.406)$
$= 1.948$	$= 1.948$

$a = 2.41$
$b = 1.95$

Question 5 continued

b) $t=20,\ h=7$

let's use the model to check this

$h^2 = at+b \Rightarrow h^2 = 2.41t + 1.94$

when $t=20$:

$h^2 = 2.41(20) + 1.94$

$h^2 = 50.14$

$h = 7.081$ which is very close to 7m

$\therefore$ the model is valid

(Total for Question 5 is 6 marks)

6.

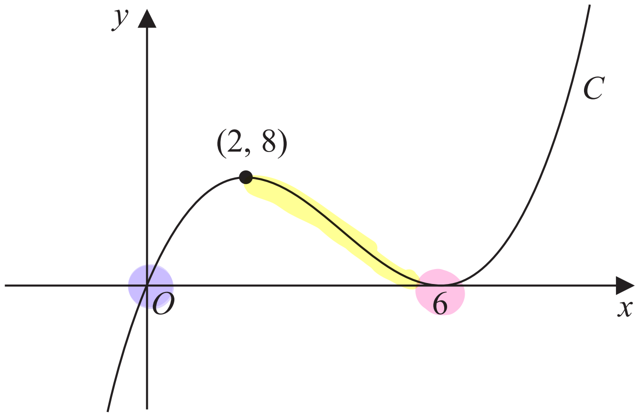


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ where $f(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$f'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

(2)

(c) Find the equation of C. You may leave your answer in factorised form.

(3)

a) This wants where the gradient/slope is negative, hence where f(x) is decreasing (graph is going downwards)

$2 < x < 6$

b) We need where the vertical line only crosses the graph once

$\{k : k < 0, k > 8\}$

Question 6 continued

c) roots are at $x=0$, $x=6$

$y = a(x-0)(x-6)^2$

2 since the curve bounces back (doesn't shoot through the root) i.e repeated root

$y = ax(x-6)^2$

We need to find the value of a
plug in the point (2, 8)

$8 = 2a(2-6)^2$

$8 = 2a(16)$

$8 = 32a$

$a = \frac{1}{4}$

$y = \frac{1}{4}x(x-6)^2$

(Total for Question 6 is 6 marks)

7. (i) Given that p and q are integers such that

$$pq \text{ is even}$$

use algebra to prove by contradiction that at least one of p or q is even.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x + y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2)

i)
Assume if at least one of p or q is even then pq is odd

p even: $p = 2k$ where k and m are integers
q odd: $q = 2m+1$

$pq = (2k)(2m+1) = 4km + 2k$
$= 2(2km + k)$
$=$ multiple of 2 $\therefore$ even

This contradicts the assumption that pq is odd

hence if at least one of p or q is even then pq is even

ii) $(x+y)^2 < 9x^2 + y^2$

$x^2 + 2xy + y^2 < 9x^2 + y^2$

$8x^2 - 2xy > 0$

$2x(4x - y) > 0$

>0 $+$

given $x<0$ $-$

P 6 9 6 0 1 A 0 1 4 4 8

Question 7 continued

So $(+)(-)(?) > 0$

must be < 0 in order to have > 0

$\Rightarrow 4x - y < 0$

$\Rightarrow y > 4x$

QED

(Total for Question 7 is 5 marks)

8.

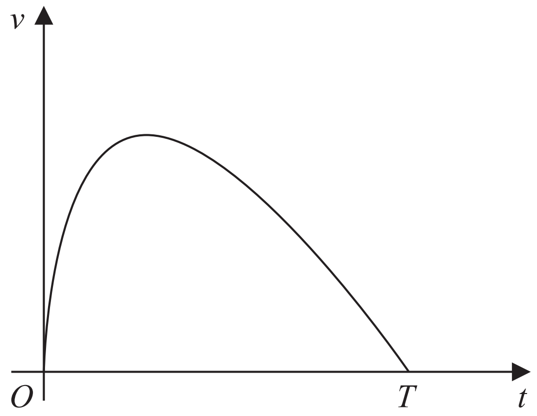


Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, $v\,\mathrm{m\,s^{-1}}$, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t)\ln(t + 1) \qquad 0 \leqslant t \leqslant T$$

where t seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of T

(1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t + 1)} - 1$$

(4)

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3)

Question 8 continued

$V = (10 - 0.4t)\ln(t+1)$

a) set $V=0$ (when reach 2^{nd} set of lights)

$\underbrace{(10-0.4t)}\underbrace{\ln(t+1)} = 0$

$10 - 0.4t = 0$
$0.4t = 10$
$t = \frac{10}{0.4}$
$= 25$

$\ln(t+1) = 0$
$e^{\ln(t+1)} = e^0$
$t+1 = 1$
$t = 0$ (this is when at 1^{st} set of lights)

$T = t = 25$ secs

b) Max speed when $\frac{dv}{dt} = 0$

use product rule to differentiate

$\frac{dv}{dt} = -0.4\ln(t+1) + (10-0.4t)\left(\frac{1}{t+1}\right)$

$\Rightarrow -0.4\ln(t+1) + \frac{10-0.4t}{t+1} = 0$

Method 1:

$\times$ by $t+1$

$\Rightarrow -0.4(t+1)\ln(t+1) + 10 - 0.4t = 0$

expand

$\Rightarrow (-0.4t - 0.4)\ln(t+1) + 10 - 0.4t = 0$

expand

$\Rightarrow -0.4t\ln(t+1) - 0.4\ln(t+1) + 10 - 0.4t = 0$

group the terms with t together on one side

$\Rightarrow 0.4t\ln(t+1) + 0.4t = 10 - 0.4\ln(t+1)$

Factorise 0.4t out

$\Rightarrow 0.4t[\ln(t+1)+1] = 10 - 0.4\ln(t+1)$

$\Rightarrow t = \dfrac{10-0.4\ln(t+1)}{0.4[\ln(t+1)+1]}$

$\Rightarrow t = \dfrac{10-0.4\ln(t+1)}{0.4\ln(t+1)+0.4}$

Divide all terms by 0.4

$\Rightarrow t = \dfrac{25-\ln(t+1)}{1+\ln(t+1)}$

$t = \dfrac{26}{1+\ln(t+1)} - \dfrac{1+\ln(t+1)}{1+\ln(t+1)}$

$t = \dfrac{26}{1+\ln(t+1)} - 1$ as required

Method 2:

Get a common denominator

$\Rightarrow \dfrac{-0.4(t+1)\ln(t+1)+10-0.4t}{t+1} = 0$

$\Rightarrow -0.4(t+1)\ln(t+1) + 10 - 0.4t = 0$

expand

$\Rightarrow (-0.4t-0.4)\ln(t+1) + 10 - 0.4t = 0$

expand

$\Rightarrow -0.4t\ln(t+1) - 0.4\ln(t+1) + 10 - 0.4t = 0$

Question 8 continued

group the terms with t together on one side

$\Rightarrow 0.4t\ln(t+1) + 0.4t = 10 - 0.4\ln(t+1)$

Factorise t out

$\Rightarrow t(0.4\ln(t+1) + 0.4) = 10 - 0.4\ln(t+1)$

$\Rightarrow t = \dfrac{10 - 0.4\ln(t+1)}{0.4\ln(t+1) + 0.4}$

$t = \dfrac{10 - \frac{4}{10}\ln(t+1)}{\frac{4}{10}\ln(t+1) + \frac{4}{10}}$ $\times 10$ $\times 10$

$t = \dfrac{100 - 4\ln(t+1)}{4\ln(t+1) + 4}$

Now use long division to get this in the required form

$$\begin{array}{r|l} & -1 \\ 4\ln(t+1) + 4 & -4\ln(t+1) + 100 \\ & -4\ln(t+1) - 4 \\ \hline & 104 \end{array}$$

$t = -1 + \dfrac{104}{4\ln(t+1) + 4}$ $\div 4$ $\div 4$

$t = -1 + \dfrac{26}{\ln(t+1) + 1}$

$t = \dfrac{26}{\ln(t+1) + 1} - 1$ as required

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Question 8 continued

Note: This was a bit hard to re-arrange in order to get the result. You could have also worked backwards from the answer to help guide you

c) $t_{n+1} = \frac{25}{\ln(t_n+1)+1} - 1$

$t_1 = 7$

i) input 7 into the calculator

press enter

$t_2 = \frac{26}{\ln(\text{ans}+1)+1} - 1$

Press enter

$t_2 = 7.443089$

Press enter again for t_3

$t_3 = 7.298$

ii) Keep pressing enter...

$t = 7.33$ seconds

(Total for Question 8 is 8 marks)

9.

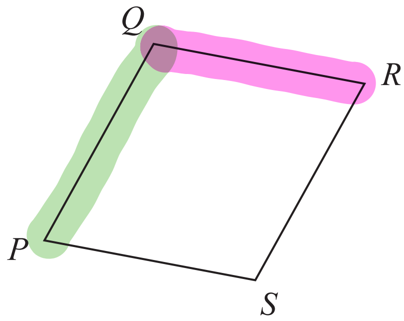


Figure 3

Figure 3 shows a sketch of a parallelogram $PQRS$.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$
- $\overrightarrow{QR} = 5\mathbf{i} - 2\mathbf{k}$

(a) show that parallelogram $PQRS$ is a rhombus.

(2)

(b) Find the exact area of the rhombus $PQRS$.

(4)

a) We are told this is a parallelogram. Only need to show 2 consecutive sides are congruent

$|\overrightarrow{PQ}| = \sqrt{2^2 + 3^2 + (-4)^2} = \sqrt{4+9+16} = \sqrt{29}$

$|\overrightarrow{QR}| = \sqrt{5^2 + (-2)^2} = \sqrt{25+4} = \sqrt{29}$

$|\overrightarrow{PQ}| = |\overrightarrow{QR}| = \sqrt{29}$ $\therefore$ rhombus

b) The area of rhombus is:

$\dfrac{\text{diagonal 1} \times \text{diagonal 2}}{2}$

We need the lengths of QS & PR

- $\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS} = -\overrightarrow{PQ} + \overrightarrow{QR} = -\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$

$\overrightarrow{PS} = \overrightarrow{QR}$

Question 9 continued

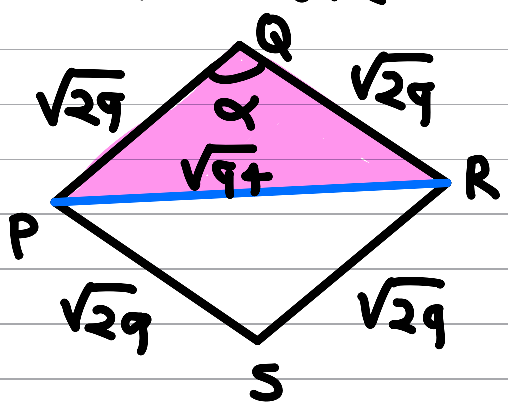
$|\overrightarrow{QS}| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9+9+4} = \sqrt{22}$

• $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ -6 \end{pmatrix}$

$|\overrightarrow{PR}| = \sqrt{7^2 + 3^2 + (-6)^2} = \sqrt{49+9+36} = \sqrt{94}$

$\text{Area} = \dfrac{\sqrt{22} \times \sqrt{94}}{2} = \dfrac{\sqrt{2068}}{2} = \dfrac{2\sqrt{517}}{2} = \sqrt{517}$ units^2

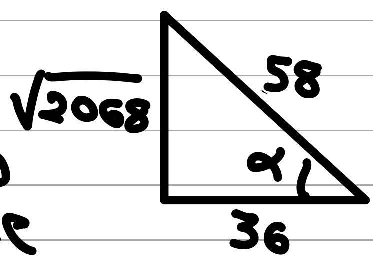
Note: we could have split the rhombus into 2 triangles and used cosine rule



$(\sqrt{94})^2 = (\sqrt{29})^2 + (\sqrt{29})^2 - 2(\sqrt{29})(\sqrt{29})\cos\alpha$

$94 = 29 + 29 - 58\cos\alpha$

$\cos\alpha = -\dfrac{36}{58}$



$\sin\alpha = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{\sqrt{2068}}{58}$

Build triangle to find $\sin\alpha$ since want exact answer

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Question 9 continued

$$\text{Area} = \frac{1}{2}\sqrt{29}\sqrt{29}\sin\alpha$$

$$= \frac{1}{2}\sqrt{29}\sqrt{29}\left(\frac{\sqrt{2068}}{58}\right)$$

$$= \frac{1}{4}\sqrt{2068}$$

$$\text{Total area} = 2\left[\frac{1}{4}\sqrt{2068}\right]$$

$$= \frac{1}{2}\sqrt{2068}$$

$$= \frac{1}{2}(2)\sqrt{517}$$

$$= \sqrt{517} \text{ units}^2$$

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10. A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands, N_b, is modelled by the equation

$$N_b = 45 + 220\,e^{0.05t}$$

where t is the number of years from the start of the study.

According to the model,

(a) find the number of bees at the start of the study,

(1)

(b) show that, exactly 10 years after the start of the study, the number of bees was increasing at a **rate** of approximately 18 thousand per year.

(3)

The number of wasps, measured in thousands, N_w, is modelled by the equation

$$N_w = 10 + 800\,e^{-0.05t}$$

where t is the number of years from the start of the study.

When $t = T$, according to the models, there are an equal number of bees and wasps.

(c) Find the value of T to 2 decimal places.

(4)

$N_b = 45 + 220e^{0.05t}$

a) let $t=0$

$N_b = 45 + 220e^{0}$

$= 45 + 220 = 265$

$= 265,000$ bees

b) rate $\Rightarrow$ differentiate

$\frac{dN}{dt} = 0 + 220(0.05)\,e^{0.05t}$

$= 11e^{0.05t}$

when $t=10$: $\frac{dN}{dt} = 18.14 \approx 18,000/\text{year}$

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Question 10 continued

c) $N_W = 10 + 800e^{-0.05t}$

$45 + 220e^{0.05t} = 10 + 800e^{-0.05t}$

$220e^{0.05t} - 800e^{-0.05t} + 35 = 0$

$220e^{0.05t} - \frac{800}{e^{0.05t}} + 35 = 0$

let $y = e^{0.05t}$

$220y - \frac{800}{y} + 35 = 0$

$200y^2 + 35y - 800 = 0$

use quadratic formula

$y = 1.829$ or $y = -1.988$

$\Rightarrow e^{0.05t} = 1.829$ or $e^{0.05t} = -1.988$

$\ln e^{0.05t} = \ln 1.829$

$0.05t = \ln 1.829$

$t = \frac{\ln 1.829}{0.05}$

$= 12.08$ years

no solution

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11.

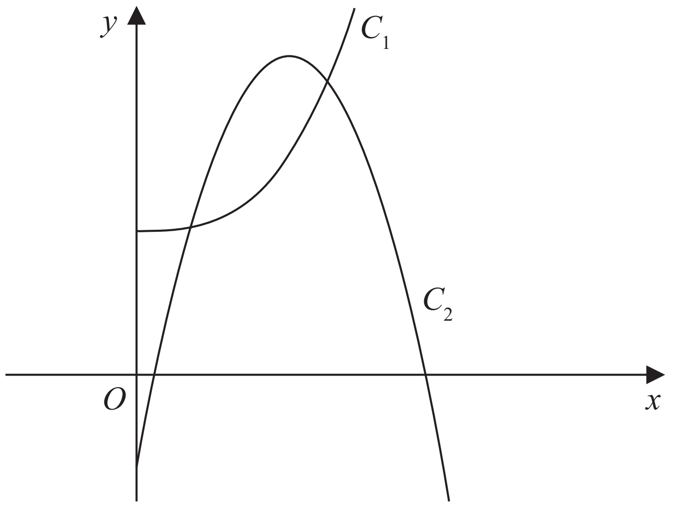

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \qquad x > 0$$

(a) Verify that the curves intersect at $x = \frac{1}{2}$

(2)

The curves intersect again at the point P

(b) Using algebra and showing all stages of working, find the exact x coordinate of P

(5)

a)

When $x = \frac{1}{2}$: $y = 2\left(\frac{1}{2}\right)^3 + 10 = \frac{41}{4}$ for C_1

When $x = \frac{1}{2}$: $y = 42\left(\frac{1}{2}\right) - 15\left(\frac{1}{2}\right)^2 - 7 = \frac{41}{4}$ for C_2

b) intersection points $\Rightarrow$ solve simultaneously

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Question 11 continued

$2x^3 + 10 = 42x - 15x^2 - 7$

$2x^3 + 15x^2 - 42x + 10 + 7 = 0$

$2x^3 + 15x^2 - 42x + 17 = 0$

We know $\frac{1}{2}$ is a solution already so let's divide by $(x - 0.5)$

$$
\begin{array}{r l}
 & 2x^2 + 16x - 34 \\
x - 0.5 \,\big) & 2x^3 + 15x^2 - 42x + 17 \\
- & \underline{2x^3 - x^2} \\
 & 16x^2 - 42x \\
- & \underline{16x^2 - 8x} \\
 & -34x + 17 \\
- & \underline{-34x + 17} \\
 & 0
\end{array}
$$

$\underbrace{(x - 0.5)}_{x = \frac{1}{2}}\underbrace{(2x^2 + 16x - 34)} = 0$

$x = \frac{1}{2}$
(we know this already)

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-16 \pm \sqrt{16^2 - 4(2)(-34)}}{2(2)}$

$= \frac{-16 \pm \sqrt{528}}{4}$

$= \frac{-16 \pm 16\sqrt{33}}{4}$

$= -4 \pm \sqrt{33}$

Question 11 continued

We can see from the graph that x is positive

$\Rightarrow x = \sqrt{33} - 4$

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12. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Show that

$$\int_1^{e^2} x^3 \ln x \, dx = ae^8 + b$$

where a and b are rational constants to be found.

(5)

Use integration by parts

$\int^{e^2} x^3 \ln x \, dx$

$u = \ln x$ $\quad \frac{dv}{dx} = x^3$

$\frac{du}{dx} = \frac{1}{x}$ $\quad v = \frac{x^4}{4}$

Formula: $uv - \int v \frac{du}{dx} dx$

$\ln x \left(\frac{x^4}{4}\right) - \int \frac{x^4}{4}\left(\frac{1}{x}\right) dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3 \, dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\frac{x^4}{4}$

$= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$

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Question 12 continued

Put the limits in

$$\left[\frac{1}{4}x^4\ln x - \frac{1}{16}x^4\right]_1^{e^2}$$

$$=\left(\frac{1}{4}(e^2)^4\ln e^2 - \frac{1}{16}(e^2)^4\right) - \left(\frac{1}{4}\ln 1 - \frac{1}{16}\right)$$

$$=\frac{1}{4}e^8(2) - \frac{1}{16}e^8 + \frac{1}{16}$$

$$=\frac{1}{2}e^8 - \frac{1}{16}e^8 + \frac{1}{16}$$

$$=\frac{7}{16}e^8 + \frac{1}{16}$$

$$a = \frac{7}{16},\ b = \frac{1}{16}$$

(Total for Question 12 is 5 marks)

13. (i) In an arithmetic series, the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}\,[2a + (n-1)d]$$

(3)

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(2)

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

(1)

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)

i)

$S_n = U_1 + U_2 + U_3 + \ldots + U_{n-1} + U_n$

$= a + (a+d) + (a+2d) + \ldots + (a+(n-2)d) + (a+(n-1)d)$ ①

Write this backwards

$S_n = (a+(n-1)d] + [a+(n-2)d] + \ldots + (a+2d) + (a+d) + a$ ②

Add ① + ②

$2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \ldots + [2a+(n-1)d] + [2a+(n-1)d]$

$2S_n = n[2a+(n-1)d]$

$S_n = \frac{n}{2}[2a+(n-1)d]$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 13 continued

ii)

a) 10, 9.20, 8.40, ...

This is arithmetic with

$a = 10$
$d = -0.80$

$S_n = 64$

$\frac{n}{2}\left[2(10) + (n-1)(-0.80)\right] = 64$

$\frac{n}{2}(20 - 0.8n + 0.8) = 64$

$n(20 - 0.8n + 0.8) = 128$

$20n - 0.8n^2 + 0.8n = 128$

$0.8n^2 - 20.8n + 128 = 0$

$\div 0.8$

$n^2 - 26n + 160 = 0$

b) $(n-10)(n-16) = 0$
$n = 10, n = 16$

c) If $n = 10$: $U_{10} = 10 + 9(-0.8) = 2.8$

If $n = 16$: $U_{16} = 10 + 15(-0.8) = -2$ (not possible)

$\therefore$ took 10 weeks to save enough

14. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$2\sin(x - 60°) = \cos(x - 30°)$$

show that

$$\tan x = 3\sqrt{3}$$

(4)

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180°$

$$2\sin 2\theta = \cos(2\theta + 30°)$$

giving your answers to one decimal place.

(4)

a) $2\sin(x-60) = \cos(x-30)$

$2(\sin x\cos 60 - \cos x\sin 60) = \cos x\cos 30 + \sin x\sin 30$

$2\left(\sin x\left(\frac{1}{2}\right) - \cos x\left(\frac{\sqrt{3}}{2}\right)\right) = \cos x\left(\frac{\sqrt{3}}{2}\right) + \sin x\left(\frac{1}{2}\right)$

$\sin x - \sqrt{3}\cos x = \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$

$2\sin x - 2\sqrt{3}\cos x = \sqrt{3}\cos x + \sin x$

$\sin x = 3\sqrt{3}\cos x$

$\frac{\sin x}{\cos x} = 3\sqrt{3}$

$\tan x = 3\sqrt{3}$

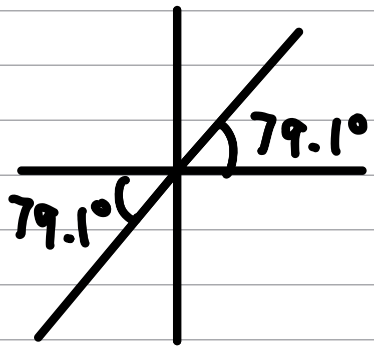
Note: could have also used the fact that $\sin 60 = \cos 30$

b) $x - 60 \rightarrow 2\theta$ so $x \rightarrow 2\theta + 60$

$\tan(2\theta + 60) = 3\sqrt{3}, \ 0 \leq \theta < 180$

Question 14 continued

$\tan^{-1}(3\sqrt{3}) = 79.1^\circ$



$0 \leq 2\theta + 60 < 420$

$2\theta + 60 = 79.1, 259.1$

$\theta = \frac{79.1 - 60}{2}, \frac{259.1 - 60}{2}$

$\theta = 9.55^\circ, 99.6^\circ$

15.

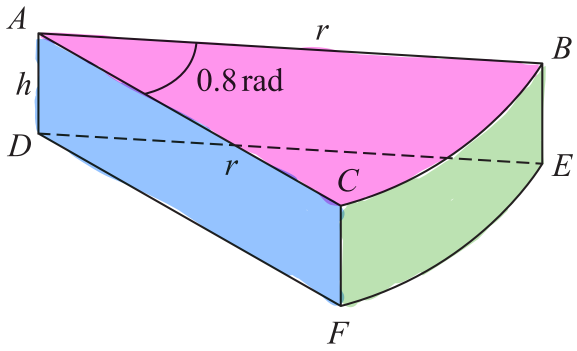


Figure 5

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face ABC is a sector of a circle with radius r cm and centre A
- angle $BAC = 0.8$ radians
- faces ABC and DEF are congruent
- edges AD, CF and BE are perpendicular to faces ABC and DEF
- edges AD, CF and BE have length h cm

Given that the volume of the toy is $240\,\text{cm}^3$

(a) show that the surface area of the toy, $S\,\text{cm}^2$, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

(2)

a) Volume = (area of cross section) x depth

$= \frac{1}{2}r^2(0.8)h$

$= 0.4r^2h$

$0.4r^2h = 240$ 1

P 6 9 6 0 1 A 0 4 2 4 8

Question 15 continued

Surface area $= S = 2\left(\frac{1}{2}r^2(0.8)\right) + 2hr + r(0.8)h$

$= 0.8r^2 + 2hr + 0.8rh$

$= 0.8r^2 + 2.8rh$ ②

We want the surface area in terms of r only. Let's use ① to achieve this

$0.4r^2h = 240$

$h = \frac{240}{0.4r^2}$

$h = \frac{600}{r^2}$

sub this into ②

$S = 0.8r^2 + 2.8r\left(\frac{600}{r^2}\right)$

$S = 0.8r^2 + \frac{1680}{r}$

b) $\frac{dS}{dr} = 1.6r - 1680r^{-2}$

Stationary when $\frac{dS}{dr} = 0$

$1.6r - 1680r^{-2} = 0$

Question 15 continued

$1.6r - \frac{1680}{r^2} = 0$

$1.6r^3 - 1680 = 0$

$r^3 = \frac{1680}{1.6}$

$r^3 = 1050$

$r = \sqrt[3]{1050} = 10.16$ cm

c) $\frac{d^2S}{dr^2} = 1.6 + 3360r^{-3}$

$= 1.6 + \frac{3360}{r^3}$

When $r = 10.16$

$\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{10.16^3}$

$= 4.803$

$> 0 \therefore$ min

16.

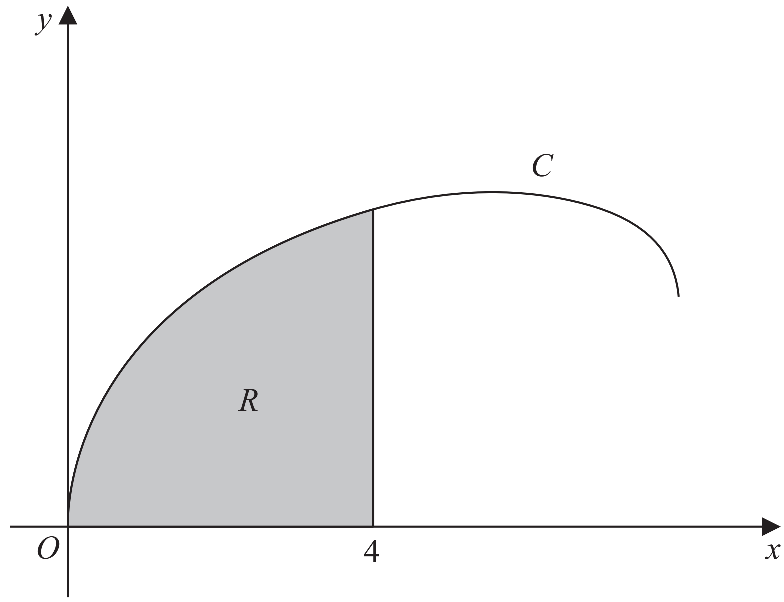


Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 8\sin^2 t \qquad y = 2\sin 2t + 3\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 6, is bounded by C, the x-axis and the line with equation $x = 4$

(a) Show that the area of R is given by

$$\int_0^a \left(8 - 8\cos 4t + 48\sin^2 t \cos t\right) \mathrm{d}t$$

where a is a constant to be found.

(5)

(b) Hence, using algebraic integration, find the exact area of R.

(4)

a) $\int y \frac{dx}{dt} \, dt$

$x = 8(\sin t)^2$

$\frac{dx}{dt} = 8(2) \sin t \cos t$

$= 16 \sin t \cos t$

Limits: $x = 0$: $8\sin^2 t = 0$

$\sin^2 t = 0$

$t = 0$

P 6 9 6 0 1 A 0 4 6 4 8

Question 16 continued

$x = 4: \quad 8\sin^2 t = 4$

$\sin^2 t = \frac{1}{2}$

$\sin t = \sqrt{\frac{1}{2}}$

$t = \sin^{-1}\sqrt{\frac{1}{2}} = \frac{\pi}{4}$

$\int 4 \frac{dx}{dt}\, dt$

becomes

$\int_0^{\frac{\pi}{4}} (2\sin 2t + 3\sin t)(16\sin t\cos t)\, dt$

$= \int_0^{\frac{\pi}{4}} (4\sin t\cos t + 3\sin t)(16\sin t\cos t)\, dt$

multiply out the brackets

$= \int_0^{\frac{\pi}{4}} (64\sin^2 t\cos^2 t + \underbrace{48\sin^2 t\cos t}_{})\, dt$

leave this (already the answer we want)

need to use double angle to deal with all of these

$\cos 2t = \begin{cases} 1 - 2\sin^2 t & (1) \\ 2\cos^2 t - 1 & (2) \end{cases}$

Re-arranging both

Question 16 continued

① $\sin^2 t = \frac{1-\cos 2t}{2}$

② $\cos^2 t = \frac{\cos 2t + 1}{2}$

Sub ① and ② back in

$$\int_0^{\frac{\pi}{4}} \left(64\left(\frac{1-\cos 2t}{2}\right)\left(\frac{\cos 2t+1}{2}\right) + 48\sin^2 t\cos t\right) dt$$

$$= \int_0^{\frac{\pi}{4}} \left(16(1-\cos 2t)(\cos 2t+1) + 48\sin^2 t\cos t\right) dt$$

expand the brackets

$$= \int_0^{\frac{\pi}{4}} \left(16(\cos 2t + 1 - \cos^2 2t - \cos 2t) + 48\sin^2 t\cos t\right) dt$$

$$= \int_0^{\frac{\pi}{4}} \left(16(1-\underbrace{\cos^2 2t}) + 48\sin^2 t\cos t\right) dt$$

use double angle again

$\cos^2 2t = \frac{\cos 4t + 1}{2}$

$$= \int_0^{\frac{\pi}{4}} \left(16\left(1-\frac{\cos 4t+1}{2}\right) + 48\sin^2 t\cos t\right) dt$$

$$= \int_0^{\frac{\pi}{4}} \left(16 - 8\cos 4t + 8 + 48\sin^2 t\cos t\right) dt$$

(Total for Question 16 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS

$$= \int_0^{\frac{\pi}{4}} (8 - 8\cos 4t + 48\sin^2 t\cos t)\, dt$$

$a = \frac{\pi}{4}$

b) $$\int_0^{\frac{\pi}{4}} (8 - 8\cos 4t + 48\cos t(\sin t)^2)\, dt$$

$$= \left[8t - \frac{8\sin 4t}{4} + 48\frac{(\sin t)^3}{3} \right]_0^{\frac{\pi}{4}}$$

$$= \left(\frac{8\pi}{4} - \frac{8\sin \pi}{4} + \frac{48(\sin \frac{\pi}{4})^3}{3} \right) - 0$$

$$= 2\pi - \frac{8(0)}{4} + 16 \left(\frac{\sqrt{2}}{2}\right)^3$$

$$= 2\pi + 16\left(\frac{2\sqrt{2}}{8}\right)$$

$$= 2\pi + 4\sqrt{2}$$